Energy partition in nonabelian gauge fields with thermodynamic interactions

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Abstract

The thermodynamic interaction at thermodynamic equilibrium in the free fermion gas is alternatively described by the coupling of particles with a scalar thermodynamic field which features a self-interaction. The gauge fields in $SU_c(3)$ symmetry are investigated with thermodynamic interactions. Six off-diagonal gluons acquire effective masses through the decolorization of thermodynamic fields using the Higgs mechanism. Together with the rise of the fermion mass in the thermal bath, this gives the partition of the thermodynamic energy under the classical limit.

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In thermodynamics, it was once a hot controversy how the temperature transformed under the Lorentz boost (for a review, see Ref.[1] and references therein). Recently, the discussion on the controversy has been still going on (e.g., see Refs.[2, 3, 4]). This controversy may be necessarily related to that how to define the proper temperature and recognize the thermodynamic interaction in a covariant framework for specific thermal systems. For fermion gases in a covariant framework, it is economic to begin with the problem in the relativistic field theory based on the following consideration. The non-linearity as well as non-inertia due to bremsstrahlung processes and multiple collisions in the thermal system is beyond the kinetics given by equations of motion for free fermions. Some dynamical degrees of freedom need to be introduced. According to the notion of field theories, the interaction is mediated by intermediate bosons. One may therefore introduce appropriate intermediate

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bosons (understood as the thermodynamic fields) to describe thermodynamic interactions for the fermion gas at thermodynamic equilibrium in the relativistic framework. To do so, it is necessary to begin with two senarios about the proper mass at thermal environment and thermodynamic equilibrium. The first senario (I) is that the mass modified by the heat carried by the system is still observed as the proper mass at the rest frame, which abides by the spirit to define the proper mass. The second senario (II) is that the thermodynamic equilibrium is Lorentz invariant, which was once adopted in Ref. [3].

The perfect fermion gas which is free of interactions is a simple system. Beyond that, it is convenient to investigate the interactions in the fermion gas by introducing the covariant derivative instead of the conventional derivative, as the interactions of fermions are fixed by the gauge invariance[5]. The exact gauge invariance does not allow the mass term of gauge fields. It is important to see whether the spontaneous symmetry breaking occurs for the thermodynamic field and then what will happen for gauge fields with thermodynamic interactions which exist for fermions with various internal degrees of freedom (charges), considering that the gauge boson acquires mass through the coupling with the field which displays the spontaneous breakdown of symmetry under the internal Lie group[6, 7]. The exploration of possible mass acquisition for gauge bosons with thermodynamic interactions is of significance, for instance, in the possible color-deconfined system of quark-gluon plasma (QGP).

Note that the system we study here is at thermodynamic equilibrium, the equipartition of thermodynamic energy will become the only soluble condition for the proposed idea. This restricts ourselves to the topic under the classical limit, and thus it is quite different from the past investigation of the screening mass in the propagator of quantized gauge fields in the thermal bath (for monographs, see Ref.[8]). The thermodynamic energy partition explored in this study may be applied to analyze the initial temperature of QGP that is important for the QGP evolution (e.g., see Refs.[9, 10]). In the following, thermodynamic interactions in the free fermion gas are described in the relativistic field theory at first. An investigation of $SU_c(3)$ gauge bosons in the interacting thermal fermion gas follows then.

The most favorable bosons are the scalar and vector ones. The vector boson couples to the non-zero conserved thermodynamic current which does not exist in the system at thermodynamic equilibrium. One may argue that the temporal component of the vector boson may exist. But such an existence of the temporal component contradicts with the

Senario (II), since the thermal current in the thermal body appears as performing the Lorentz boost. Consequently, only the scalar boson mediates the thermodynamic interaction. The coupling of the scalar boson with the fermion is equivalent to the heating which increases the fermion mass. This requires the imaginary mass of the scalar boson, based on the knowledge of the mean-field treatment. The boson with the imaginary mass is however unphysical, and the self-interaction has to be introduced so as to obtain the real mass. The spirit in doing so is similar to that for the pseudoscalar field[11] and for the Higgs scalar field[6].

The scalar boson field, noted as the thermodynamic field below, is described by the following Lagrangian

$$\mathcal{L}_\tau = \frac{1}{2}(\partial_\mu \phi_\tau \partial^\mu \phi_\tau - \mu_\tau^2 \phi_\tau^2) - \frac{\lambda}{4}\phi_\tau^4 \tag{1}$$

where μ_τ and λ are the constants with $\mu_\tau^2 < 0$ and $\lambda > 0$. Yet, we need to obtain the real mass of ϕ_τ. The vacuum solution of field ϕ_τ is obtained as ${\phi_\tau^0}^2 = -\mu_\tau^2/\lambda$, by minimizing the potential. The constant vacuum field is symmetry-breaking under the reflection of internal field space. Substituting

$$\phi_\tau = \phi_\tau^0 + \phi = \sqrt{-\mu_\tau^2/\lambda} + \phi \tag{2}$$

into Eq.1, it becomes

$$\mathcal{L}_\tau = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m_\tau^2 \phi^2) - \lambda \phi_\tau^0 \phi^3 - \frac{\lambda}{4}\phi^4 \tag{3}$$

where the real mass of ϕ is obtained to be $m_\tau = \sqrt{-2\mu_\tau^2}$. Now, the vacuum of the thermodynamic field is actually redefined and is symmetric under field reflection, while the reflection symmetry of $\mathcal{L}_\tau$ is broken due to the term $-\lambda\phi_\tau^0\phi^3$. The scalar thermodynamic field defined here is in form similar to the Higgs field, and its spontaneous-symmetry-breaking property under internal Lie groups can be found in Refs.[6, 7, 11].

The total Lagrangian which contains the fermion, thermodynamic field, and the Yukawa coupling between them is

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m - g_\tau \phi_\tau)\psi + \mathcal{L}_\tau \tag{4}$$

where ψ is the Dirac spinor, and m is the proper mass of fermion. In homogeneous thermal matter, the scalar field ϕ is given as

$$\phi = \frac{1}{m_\tau^2}(-g_\tau \bar{\psi}\psi - 3\lambda \phi_\tau^0 \phi^2 - \lambda \phi^3) \tag{5}$$

In an alternative description for the thermodynamic interaction, the real-mass boson does not always suggest the definite existence of the boson. Actually, a scalar boson which serves the role like Higgs bosons is not found yet. Alternatively, one may take the boson mass m_τ as large enough. In this way, the field ϕ will be as small enough. This treatment is consistent with the equipartition theorem, as seen below. Now, the thermal heat of the system may just be determined by the term $g_\tau \bar{\psi}\phi_\tau^0\psi$ in Eq.4. The homogeneous thermodynamic equilibrium of the fermion system can be elaborated by the coupling with the isotropic field vacuum ϕ_τ^0. Thus, it is actually the broken vacuum of the scalar field that plays the very role, in analogy to that of the Higgs field in gauge theories[6, 7].

The potential $g_\tau\phi_\tau^0$ is a homogeneous quantum for all particles in the mean-field approximation, which indicates the equipartition theorem that holds for the classical limit is the unique solvable condition for the problem. The fermion mass measured at the rest frame at the non-relativistic limit is thus

$$m^* = m + g_\tau\phi_\tau^0 = m + \frac{3}{2}T_0 \tag{6}$$

The derivation of Eq.6 implies that it is reasonable to take m_τ as large enough. Otherwise, it will lead to the inequality $m^* \neq m + 3/2T_0$ for finite m_τ not large enough. The proper temperature T_0 in the free fermion gas, which is proportional to the modification of the proper mass of fermions as given in Eq.6, is a Lorentz scalar, consistent with the Senario (I). It may be straightforward to work out various Lorentz transformations for the apparent temperature based on the Lorentz structure of Lagrangian.

The thermodynamic interaction has been alternatively described through the dynamic scalar boson coupling in the free fermion gas in above. Now, let's investigate the more realistic system, the interacting fermion gas. The interaction forms of the interacting fermions are fixed by the gauge invariance[5]. The applications of gauge theories can be found, for instance, in Refs.[7, 12]. In the following, we consider the thermodynamic interaction in the color $SU_c(3)$ symmetry. The interactions in the Lagrangian for thermal fermions are introduced by replacing the partial derivative ∂_μ with the covariant derivative $D_\mu = \partial_\mu - igA_\mu$ with g the coupling constant. The gauge field is expressed with respect to the generators λ in matrix form as $A_\mu = \lambda^\alpha A_\mu^\alpha/2$, with $\alpha = 1, \cdots, 8$ in $SU_c(3)$ symmetry. The classical Lagrangian

involving the gauge field can be given as

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma_\mu D^\mu - m - g_\tau\phi_\tau)\psi \\ &+Tr\{\frac{1}{2}[(D_\mu\phi_\tau)^\dagger D^\mu\phi_\tau - \mu_\tau^2(\phi_\tau^\dagger\phi_\tau)\} \\ &-\frac{\lambda}{4}(Tr[\phi_\tau^\dagger\phi_\tau])^2 - \frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha\mu\nu}\end{aligned} \tag{7}$$

where the trace is over the internal color space. Mathematically, there can be the terms linear in $Tr\phi_\tau$, $Tr\phi_\tau^3$, and $Tr(\phi_\tau^\dagger\phi_\tau)^2$. Assuming the reflection symmetry for the thermodynamic field $\phi_\tau \to -\phi_\tau$, the terms having the odd powers are ruled out. Without involving the term linear in $Tr(\phi_\tau^\dagger\phi_\tau)^2$, one may obtain the field vacuum

$$\phi_\tau^0 =< \sqrt{Tr(\phi_\tau^\dagger\phi_\tau)} >_0= \sqrt{-\mu_\tau^2/\lambda} \tag{8}$$

which is a natural extension of the vacuum of $O(1)$ symmetry given by Eq.4. In this way, it is simple to perform the rotation in the internal space without adding any additional assumption on the vacuum value of thermodynamic fields. In order to keep the hermitian constraint for the Lagrangian, the field ϕ_τ should be hermitian, i.e. $\phi_\tau^\dagger = \phi_\tau$. The gauge field strength tensor is

$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + gf^{\alpha\beta\gamma}A^\beta_\mu A^\gamma_\nu \tag{9}$$

with $f^{\alpha\beta\gamma}$ the antisymmetric structure constants. The covariant derivatives for the fermion and thermodynamic field are defined, respectively

$$\begin{aligned}D_\mu\psi &= (\partial_\mu - igA^\alpha_\mu\lambda^\alpha/2)\psi \\ D_\mu\phi_\tau &= \partial_\mu\phi_\tau - i\frac{g}{2}[A^\alpha_\mu\lambda^\alpha, \phi_\tau]\end{aligned} \tag{10}$$

Under the gauge transformation

$$\begin{aligned}&\psi' = S^{-1}\psi,\ \phi'_\tau = S^{-1}\phi_\tau S \\ &A'_\mu = S^{-1}A_\mu S + \frac{i}{g}S^{-1}\partial_\mu S\end{aligned} \tag{11}$$

where $S = \exp[i\theta^\alpha(x)\lambda^\alpha/2]$ with $\theta^\alpha(x)$ the gauge functions, the Lagrangian (7) is invariant.

Because thermodynamic interactions exist for fermions with various charges (colors), the hermitian matrix of thermodynamic fields is not required to be diagonalized. Similar to the non-interacting case, the apparent consequence resulted from the thermodynamic interaction

is the modification of the fermion mass. We therefore need to perform the diagonalization for the ϕ_τ. The hermitian matrix can be diagonalized through the unitary transformation as follows

$$\phi_\tau = \exp[i\frac{\xi^\alpha\lambda^\alpha}{2}] \begin{pmatrix} \phi_{\tau 1} & & \\ & \phi_{\tau 2} & \\ & & \phi_{\tau 3} \end{pmatrix} \exp[-i\frac{\xi^\alpha\lambda^\alpha}{2}] \tag{12}$$

In diagonalized form, the ϕ_τ vacuum quantity is $\phi_\tau^0 = \sqrt{\sum_i \phi_{\tau i}^{02}}$, and one may choose any of following forms to express the vacuum in the tensor form

$$v = \begin{pmatrix} \phi_\tau^0 & & \\ & 0 & \\ & & 0 \end{pmatrix}, \begin{pmatrix} 0 & & \\ & \phi_\tau^0 & \\ & & 0 \end{pmatrix}, \begin{pmatrix} 0 & & \\ & 0 & \\ & & \phi_\tau^0 \end{pmatrix} \tag{13}$$

Any one of tensor vacua can be rotated to other forms by the symmetry-breaking generator λ^β such as performing $\lambda^\beta v_{\beta\beta}\lambda^\beta$. The thermodynamic field can be parametrized based on the given vacuum. The parametrization is simple as knowing that the diagonal elements in Eq.12 are the real quantities. We just need replace the medium matrix in Eq.12, and it is

$$\phi_\tau = \exp[i\frac{\xi^\alpha\lambda^\alpha}{2}](u+v)\exp[-i\frac{\xi^\alpha\lambda^\alpha}{2}] \tag{14}$$

where u is the same as v but the nonzero vacuum quantity is substituted by the field ϕ. In order to give the modification to the fermion mass of all colors by thermodynamic interactions, three forms of vacuum are all needed. Thus the Lagrangian should involve thermodynamic fields with all of three vacuum choices, and for simplicity it is not rewritten here.

To give the modification to the fermion mass, the exponential factors in Eqs.12,14 should be eaten. Moreover, massless Goldstone bosons[11], appearing as the symmetry of vacuum is broken under the present $SU_c(3)$ group, should be exorcised using the Higgs mechanism[6]. In the parametrized expression of ϕ_τ given in Eq.14, Goldstone bosons are described by tensors $[\lambda^\beta, v]\xi^\beta$ which satisfy the relations $\lambda^\beta v \neq 0$ and $v\lambda^\beta = (\lambda^\beta v)^\dagger \neq 0$ with M (< 8) choices for β. Both tasks are accomplished simultaneously as we perform the gauge transformation given in Eq.11 under the unitary gauge $\theta^\alpha = \xi^\alpha$.

A straightforward consequence of the coupling between the gauge and thermodynamic fields is the generating of gauge boson masses, abiding by the Higgs mechanism. The mass term of the gauge field is obtained from the term $(D_\mu\phi)^\dagger D^\mu\phi$ in Eq.7. To acquire the effective gluon mass, we just need sum up the terms linear in v^2 contained in $(D_\mu\phi)^\dagger D^\mu\phi$ over three

vacua as

$$m_\alpha^2 A_\mu^\alpha A^{\alpha\mu} = -\frac{g^2}{4}\sum_v Tr\{[\lambda^\alpha A_\mu^\alpha, v][\lambda^\sigma A_\mu^\sigma, v]\} \tag{15}$$

with $\alpha, \sigma = 1, \cdots, 8$. Thus, the effective gluon masses are

$$m_\alpha = g\phi_\tau^0, \text{ for } \alpha \neq 3, 8 \tag{16}$$

For A_μ^3 and A_μ^8, they keep massless.

An important procedure adopted above is the diagonalization which is actually the decolorization of thermodynamic fields. Through the decolorization of thermodynamic fields, six off-diagonal gluons acquire effective masses. For the electromagnetic interaction in U(1) symmetry, the coupling of fermions with the thermal bath is through the exchange of the neutral thermodynamic field, according to the hermitian constraint on the Lagrangian. There is no coupling between the gauge boson and thermodynamic field in $U(1)$ symmetry where there is the relation $D_\mu\phi_\tau = \partial_\mu\phi_\tau$ for the thermodynamic field as given by Eq.10, and this is actually quite general for the symmetry in abelian groups. No photo can acquire the effective mass in the thermodynamic field. This is consistent with the fact that the equipartition theorem is not able to be applied to the photos for which the quantum effect is always important. Similar to photos, two diagonal colorless gluons which relate to the color neutral current keep massless. The mass acquisition of off-diagonal gauge bosons in other $SU(N)$ symmetries with thermodynamic interactions may be on the analogy of the present case in $SU_c(3)$ symmetry. As known from above, the effective gluon mass is linear in the proper temperature if the temperature is defined by $g_\tau\phi_\tau^0$ as in Eq.6.

The equipartition theorem is appropriate for the limit that particles may pass through all possible states. As the particles are in high degeneracy or the quantum effect is important, particles are not able to pass through all states. Thus, the quantity $g_\tau\phi_\tau^0$ provided in the mean-field approximation exists on the sense that the equipartition theorem is appropriate for the thermal system. For the gauge bosons, the acquisition of the effective mass is equivalent to the partition of the thermodynamic energy at the classical limit. So, the classical Boltzmann distribution can be applied to the quarks and six off-diagonal gluons for some cases that are consistent with the senario (I), whereas for the photon and colorless gluons the quantum effect is always important. If the equipartition theorem is applicable, the effective masses of colored gluons is obtained as $m_\alpha = 3T_0$ for the ultra-relativistic case, and same effective quark

masses are obtained considering the interaction is weak due to asymptotic freedom. This case may eventually exist in an extremely hot QGP with a high fugacity where the quantum effect in distributions of colored partons may be negligible[9]. However this is not simply to say that it has the relation $g_\tau = g$ since here the interactions in QGP is neglected. In addition to considering photons and diagonal gluons always obey the Bose distribution, one is able to carry out the partitioned energy for each parton in the sense of statistical average. As the quantum effect in parton distributions becomes important during the cooling, the concept of the equipartition is not applicable since the universal mean quantity $(g_\tau \phi_\tau^0)$ does not exist again.

In summary, the thermodynamic interaction in fermion gases is alternatively described by the exchange of a scalar boson, based on notions of the relativistic field theory and the senario for thermodynamic equilibrium. The scalar boson field (thermodynamic field) features a self-interaction like the Higgs scalar field. The gauge fields in $SU_c(3)$ symmetry are investigated with thermodynamic interactions. The coupling between the gauge and thermodynamic fields results in the generation of the effective mass of six off-diagonal gluons, which is realized through the decolorization of thermodynamic fields using the Higgs mechanism. Two diagonal gluons still keep massless. Apart from the nonabelian case, the mass acquisition does not take place for the abelian gauge boson in thermodynamic interactions. The modification of the fermion proper mass in the thermal bath is equivalently described by the coupling with the symmetry-breaking vacuum of the scalar thermodynamic field. The effective masses acquired are the partitioned thermodynamic energies that are observed to have the same property of the proper mass in the center of mass frame. The investigation holds for the case that the quantum effect together with the interaction of the system is weak.

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